Deformation of the Earth's Surface by Local Mass Loading and Uploading Effects

Marcel Mojzeš[1], *Pavol Kollar*[1], *Miloš Vaľko*[2]

Slovak University of Technology in Bratislava, West Bohemian University in Pilsen

e-mail: marcel.mojzes@stuba.sk milos.valko@wbupl.cz

Abstract. The mass of a building influences a deformation of the Earth's surface up to long distance from area of realization. The elastic and rheological effects of settlement are modeled by different methods presented in the many scientific papers. More deeply knowledge of mechanical properties of materials and modern computer equipment allow construct very precisely model reaction of the Earth surface on loading effects in time. Geodesists and geophysicists need to know this laws of deformation and theirs time evolution for design of control geodetic networks and for analysis of repeated control measurements of horizontal and vertical time series. The presentation contents the mathematical modeling of the Earth elastic surface deformation caused by loading and unloading effects caused by a mass.

Keywords: mass loading, compressibility of soil, elastic settlement of earth's surface

1 Introduction

A stress increase caused by the construction of buildings or other loads compresses soil layers. The compression is caused by (a) deformation of soil particles, (b) relocations of soil particles, and (c) expulsion of water or air from void spaces. In general, the soil settlement caused by load may be divided into three broad categories (Das, 2010):

1. Elastic settlement, which is caused by elastic deformation of dry soil and of moisture and saturated soils without any

changes in moisture content. Elastic settlement calculations generally are based on equations derived from the theory of elasticity.

2. Primary consolidation settlement, which is the result of a volume change in saturated cohesive soil because of expulsion of the water that occupies the void spaces.

3. Secondary consolidation settlement, which is observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

This paper presents the fundamental principles for estimating the elastic and consolidation settlements of soil layers under superimposed loadings.

The total settlement of an Earth's surface can then be given as,

$$S_T = S_C + S_S + S_e, \tag{1}$$

where S_T is total settlement, S_c is primary consolidation settlement, S_s is secondary consolidation settlement, S_e is elastic settlement.

When buildings are constructed on very compressible clays, the consolidation settlement can be several times greater than the elastic settlement.

2 Elastic settlement

Elastic, or immediate, settlement of building (S_e) occurs directly after the application of a load without a change in the moisture content of the soil. The magnitude of the contact settlement will depend on the flexibility of the building and the type of material on which it is resting.

In paper (Kollár et al., 2010) the relationship for determining the increase in stress (which causes elastic settlement) due to the application of circular load was based on following assumptions:

- The load is applied at the ground surface.

- The loaded area is flexible.
- The soil medium is homogeneous, elastic, isotropic, and extends to a great depth.

2.1 Circular loading

Let we start with study of deformation surface boundary of half-space, which is on circle area of radius R loaded by uniformly loading of permanent intensity $p_1 = const$.

The displacement function is necessary construct for 2 areas: area of loading $0 \leq r < R$ $(0 \leq \rho < 1)$ and for non loaded area $R \leq r < \infty$ $(1 \leq \rho < \infty)$ for loading intensity. For area $0 \leq r < R$ we have,

$$\nu_z(\rho) = \frac{4 \cdot (1 - \mu_z^2)}{\pi E_z} \left[\int_0^{\rho} \frac{\overline{\rho}}{\rho} \mathbf{K} \left(\frac{\overline{\rho}}{\rho}, \frac{\pi}{2} \right) p_1 R \mathrm{d}\overline{\rho}, \right] \tag{2}$$

and for area $1 \leq \rho < \infty$,

$$\nu_z(\rho) = \frac{4 \cdot (1 - \mu_z^2)}{\pi E_z} \int_0^{1} \frac{\overline{\rho}}{\rho} \mathbf{K} \left(\frac{\overline{\rho}}{\rho}, \frac{\pi}{2} \right) p_1 R \mathrm{d}\overline{\rho}, \tag{3}$$

where $S_e = \nu_z$.

Solving of integrals (2) and (3) using substitution

$$\begin{aligned} \frac{\overline{\rho}}{\rho} &= \xi \to \overline{\rho} = \rho\xi, & \mathrm{d}\overline{\rho} &= \rho \mathrm{d}\xi, \\ \frac{\rho}{\overline{\rho}} &= \eta \to \overline{\rho} = \rho\eta^{-1}, & \mathrm{d}\overline{\rho} &= -\frac{\rho}{\eta^2}\mathrm{d}\eta, \end{aligned} \tag{4}$$

is given by Ryžik and Gradštein 1963 (1963, page 641) in closed following form: for $0 \leq \rho \leq 1$

$$\nu_z(\rho) = \frac{4 \cdot (1-\mu_z^2)Rp_1}{\pi E_z} \cdot \left\{ \rho^2 \left[\mathbf{E}(\xi, \frac{\pi}{2}) - (1-\xi^2)\mathbf{K}(\xi, \frac{\pi}{2}) \right]_0^1 + \rho \left[\frac{\mathbf{E}(\eta, \frac{\pi}{2})}{\eta} \right]_1^\rho \right\} = \frac{4 \cdot (1-\mu_z^2)Rp_1}{\pi E_z} \cdot \mathbf{E}(\rho, \frac{\pi}{2}) \quad (5)$$

and for $1 \leq \rho \leq \infty$

$$\nu_z(\rho) = \frac{4 \cdot (1-\mu_z^2)Rp_1}{\pi E_z} \cdot \frac{1}{\rho} \left\{ \rho^2 \left[\mathbf{E}(\xi, \frac{\pi}{2}) - (1-\xi^2)\mathbf{K}(\xi, \frac{\pi}{2}) \right]_0^{\frac{1}{\rho}} \right\} = \frac{4 \cdot (1-\mu_z^2)Rp_1}{\pi E_z} \cdot \rho \left\{ \left[\mathbf{E}(\frac{1}{\rho}, \frac{\pi}{2}) - (1-\frac{1}{\rho^2})\mathbf{K}(\frac{1}{\rho}, \frac{\pi}{2}) \right] \right\} \quad (6)$$

where $\mathbf{K}(k, \frac{\pi}{2})$ and $\mathbf{E}(k, \frac{\pi}{2})$ are fully elliptic integrals of first and second type given by

$$\mathbf{K}(k, \frac{\pi}{2}) \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\omega}{\sqrt{1-k^2\sin^2\omega}} = \frac{\pi}{2} \left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1.3}{2.4}\right) k^4 + \cdots + \left[\frac{(2n-1)!!}{2^n n!}\right] k^{2n} + \cdots \right]$$

$$\mathbf{E}(k, \frac{\pi}{2}) \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\omega}\,\mathrm{d}\omega = \frac{\pi}{2} \left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1.3}{2.4}\right) \frac{k^4}{3} - \cdots - \left[\frac{(2n-1)!!}{2^n n!}\right] \frac{k^{2n}}{2n-1} - \cdots \right] \quad (7)$$

and $k = \rho, \frac{1}{\rho}$

Deformation surface constructed using (6) and (7) is visible in Fig. 1. Some discrete values of function (6) are given in Tab. 1.

Table 1. *Discreet values of dimensionless displacements boundary of half-space is visible on Fig. 1*

ρ	0	0.4	0.8	1.2	1.6	2.0	2.4
$\frac{\nu_z(\rho)}{\frac{4\cdot(1-\mu_z^2)Rp_1}{\pi E_z}}$	1.5708	1.5059	1.2764	0.7365	0.5193	0.4063	0.3349

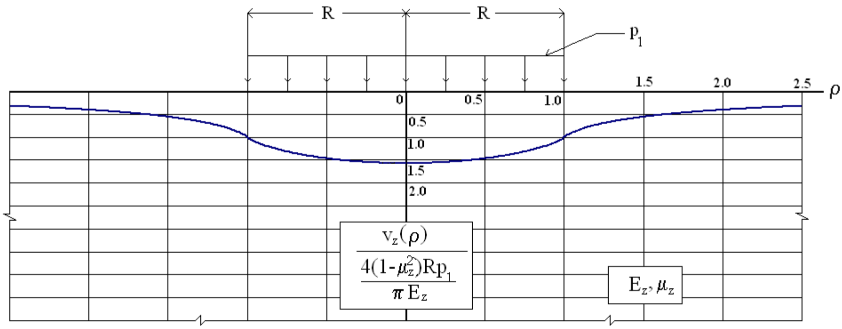

Figure 1. *Deformation surface boundary of half-space caused by circular loading*

2.2 Rectangular loading

Boussinesq's solution also can be used to calculate the vertical stress caused by a rectangular loaded area. The loaded area is located at the ground surface with length $a_2 - a_1$ and width $b_2 - b_1$. The uniformly distributed load per unit area is equal to p. The settlement of the earth's surface may be computed by formula

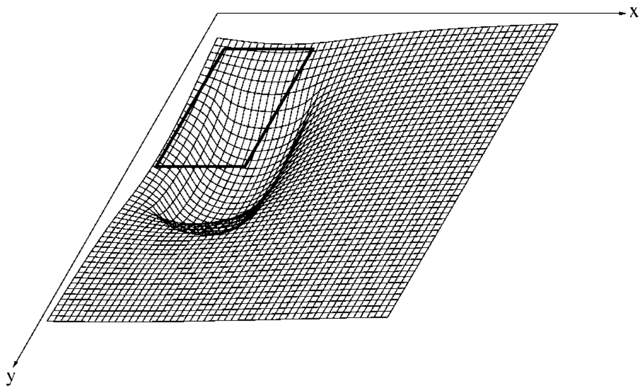

Figure 2. *Rectangular loading*

$$\nu_z(x,y) = \frac{(1-\mu_z^2)p}{\pi E_z}\left[(y-b_1)\ln\frac{x-a_1+\sqrt{(x-a_1)^2+(y-b_1)^2}}{x-a_2+\sqrt{(x-a_2)^2+(y-b_1)^2}}+\right.$$
$$(y-b_2)\ln\frac{x-a_2+\sqrt{(x-a_2)^2+(y-b_2)^2}}{x-a_1+\sqrt{(x-a_1)^2+(y-b_2)^2}}+$$
$$(x-a_1)\ln\frac{y-b_1+\sqrt{(x-a_1)^2+(y-b_1)^2}}{y-b_2+\sqrt{(x-a_2)^2+(y-b_2)^2}}$$
$$\left.(x-a_2)\ln\frac{y-b_2+\sqrt{(x-a_2)^2+(y-b_2)^2}}{y-b_1+\sqrt{(x-a_1)^2+(y-b_1)^2}}\right] \quad (8)$$

where earth's surface is homogeneous, isotropic and perfect elastic body (half-space) of Boussinesq's type. The material properties are elastic of the earth's surface characterized with Young's modulus of elasticity E_z (in GPa) and Poisson ratio μ_z (dimensionless). The loading of half-space is permanent, uniform, normal loading. The analytical solutions are based on Green loading function. The rectangular loading is presented on Fig. 2.

Table 2. *Differences between computed and controlled measurements by precise leveling*

Point	Model [mm]	Levelling [mm]	Difference [mm]
PB-2	0.018	0.008	+0.010
DB-5	0.023	0.018	+0.005
DB-4	0.022	0.015	+0.007
PB-1	0.033	0.025	+0.007
PB-11	0.017	0.008	+0.001
PB-10	0.017	0.007	+0.009
PB-9	0.023	0.020	+0.003
PB-7	0.017	0.021	−0.005
PB-1	0.013	0.005	+0.008
PB-3	0.014	0.014	+0.000
PB-4	0.021	0.012	+0.009
PB-5	0.013	0.012	+0.000
PB-6	0.021	0.031	−0.010
PB-8	0.024	0.027	−0.003

2.3 Practical experiment

The practical experiment was applied to water dam Gabčíkovo. The water dam Gabčíkovo consists from two pits. The first pit is water dam with dimensions 382 m x 214 m x 33 m and second pit is shipping chamber with dimensions 429 m x 188 m x 23 m (see Fig. 3). The material from the pit was transported from 1984 to 1986. The vertical movements of the Earth's surface were controlled by precise leveling on the special monument points (details see Janek, 1998). The results of computed deformation of Earth's surface are presented on Fig. 3. The results of the earth's surface uplift controlled by precise leveling in the 1984 (start) and 1986 (end) are presented in Fig. 4 and Tab. 2.

The standard deviation of the differences was computed by formula

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i^{mod} = x_i^{niv} - \mu^2} = 0.006\,\text{mm} \tag{9}$$

where

$$\mu = \frac{1}{n}\sum_{i=1}^{n}(x_i^{mod} = x_i^{niv}) = 0.003\,\text{mm} \tag{10}$$

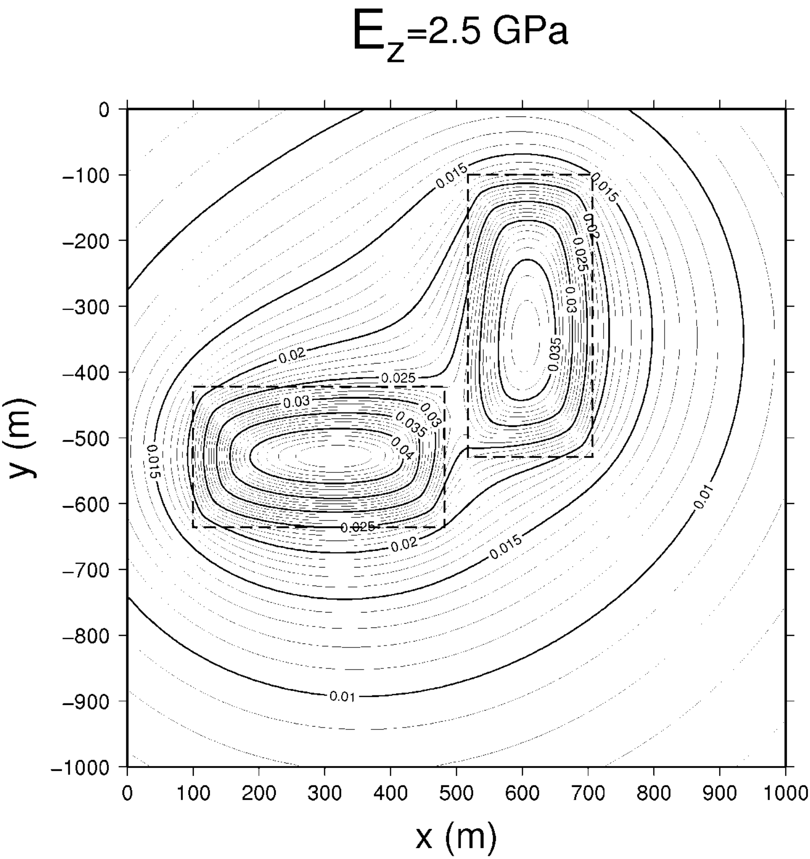

Figure 3. *Uplift effect of the earth's surface in the area of Gabčíkovo*

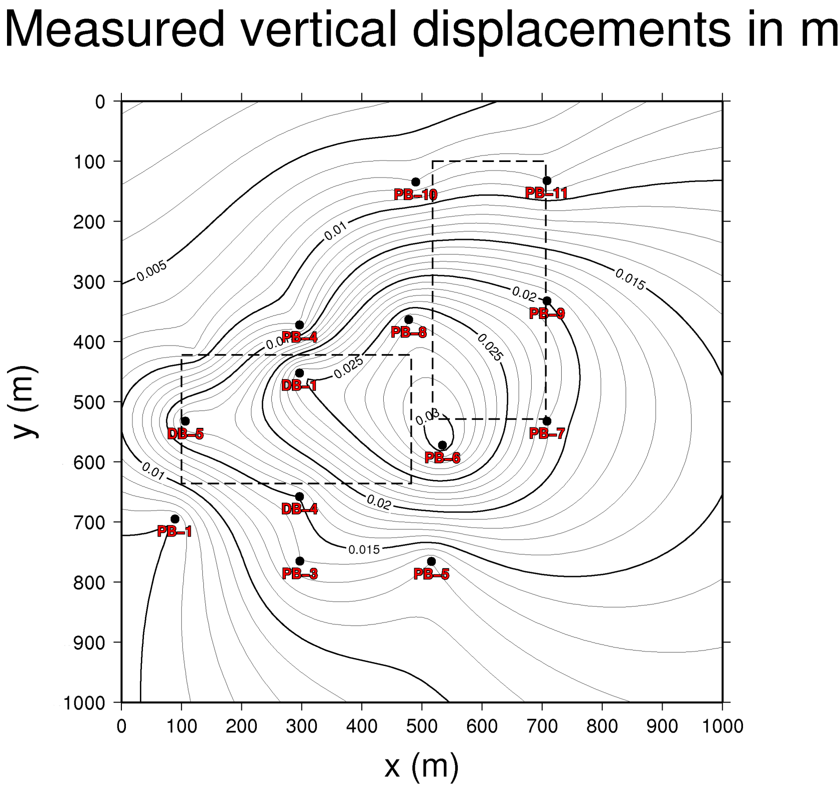

Figure 4. *Measured vertical displacement by precise leveling (in meters)*

3 Conclusion

This paper presents the relationships for determining vertical stress at a point due to application of the circular and rectangular loading. These relationships are derived by integration of Boussinesq's equation for point load. The equations presented in this paper are based entirely on the principles of the theory of elasticity; however, one must be realize the limitations of these theories when they are applied to a soil medium. These is because soil medium, in generally, are not homogeneous, perfectly elastic, and isotropic. Hence, some deviations from the theoretical calculations can be expected. On the basis of these results, presented in the paper, it could expect a difference of ±25% to ±30% between theoretical estimates and actual field values.

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